MA111 - Engineering Mathematics - II
Problem Sheet - 9

Wronskian, Nonhomogeneous ODEs and Solution by Variation of Parameters

1. Prove or disprove that the following pair of functions are basis of a linear ODE.

   (a) $\cos 5x, \sin 5x$

   (b) $x^k \cos(\ln x), x^k \sin(\ln x)$

   (c) $x, 1/x$

   (d) e^x, e^{x-1}.

2. Find the general solution

   (a) $y'' + 4y' + 4y = e^{-x} \cos x$

   (b) $(3D^2 + 27I)y = 3 \cos x + \cos 3x$

   (c) $(D^2 + 2D + \frac{3}{4}I)y = 3e^x + \frac{a}{2}x$.

3. Solve the IVPs

   (a) $y'' + 3y = 18x^2, y(0) = -3, y'(0) = 0$

   (b) $(x^2 D^2 - 3xD + 3I)y = 3 \ln x - 4, y(1) = 0, y'(1) = 1$.

4. Solve

   (a) $y'' - 4y' + 5y = e^{2x} \operatorname{cosec} x$

   (b) $(D^2 - 7I)y = \frac{1}{\cos hx}$.

5. Solve $y'' - 2y' + y = 35x^{3/2}e^x + x^2$ by using the method of variation of parameter.

6. Show that the following set of functions forms a basis of an linear ODE.

   (a) $\{1, x^2, x^4\}$

   (b) $\{1, e^{-x} \cos 2x, e^{-x} \sin 2x\}$.

7. Are the following functions linearly independent?

   (a) $x^2, \frac{1}{x^2}, 0$ on $(0, \infty)$

   (b) $e^x \cos x, e^x \sin x, e^x$ on $\mathbb{R}$

   (c) $\cos^2 x, \sin^2 x, \cos^2 x \sin^2 x$ on $\mathbb{R}$.

*****